

Continued fraction matrix representation of response functions in multicomponent systems

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The continued fraction representation of response functions is developed for a set of dynamical variables. Various approximation schemes are possible in which the frequency-moment sum rules appear explicitly. This formalism is applied to light scattering in two-component strongly coupled plasmas as an illustrative example.

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Response functions are indispensable for describing properties of many-body systems and can directly be associated with experimental observables. A general theoretical scheme for the calculation of response functions is provided by the Mori approach [1], where the underlying time-correlation functions are phrased in terms of memory function equations. The Mori formalism finds wide applications, ranging from nuclear physics [2] to superconductivity [3] to liquid theory [4] and glass transitions [5]. As originally shown by Mori [6], and later developed by Lee [7], a correlation function can be expressed in Laplace space in the form of a continued fraction (CF) whose terms depend uniquely on the sum rules, which are the frequency moments of the correlations functions. The CF representation yields transparent schemes of approximation with exact satisfaction of several sum rules. It has been applied, for example, to spin systems [8], one-component plasmas [9], and ergodic theory [10]. Recently, a comprehensive review of these ideas has been given by Balucani, Lee, and Tognetti [11].

Despite the considerable interest in the CF representation, most applications have been towards properties that involve a single dynamical variable. Here, we formulate the CF representation in a matrix form that includes a set of independent dynamical variables. Such a representation can be applied to single-component systems with important couplings between several dynamical variables, or generally to multicomponent systems with multiple important dynamical variables. Following Mori, we manipulate an n -dimensional vector of dynamical variables and, through an orthogonalization process similar to Lee's [7,12], a response matrix is constructed that contains all coupled response functions derivable from the n dynamical variables. As an illustrative example, we employ the formalism to model the Thomson scattering cross section of a dense two-component plasma.

Consider a many-body system subjected to a weak external perturbation. If the perturbation induces fluctuations in a set of dynamical variables A_1, A_2, \dots, A_n ($n \geq 1$) of interest [13], which we include in a vector \mathbf{A} , then the response function can be written as

$$\tilde{\chi}(t) = \beta \theta(t) (\mathbf{B}, \dot{\mathbf{A}}(t)), \quad (1)$$

where the applied external field couples to the dynamical

variables $\mathbf{B} \equiv (B_1, B_2, \dots, B_m)$ ($m \geq 1$). Here $\beta = 1/k_B T$ is the inverse temperature and θ is the Heaviside function and $\dot{\mathbf{A}}(t) = d\mathbf{A}/dt$. We also define the correlation matrix between two vectors \mathbf{A} and \mathbf{B} as the matrix of the correlation functions (A_i, B_j) , i.e., $(\mathbf{A}, \mathbf{B}) = ((A_i, B_j))_{i,j}$. In the quantum case, (A, B) is the Kubo product [14]

$$(\mathbf{A}, \mathbf{B}) \equiv \frac{1}{\beta} \int_0^\beta d\lambda \langle \exp(\lambda H) \mathbf{A} \exp(-\lambda H) \mathbf{B}^\dagger \rangle, \quad (2)$$

where the dagger denotes Hermitian conjugation; in the classical limit, Eq. (2) reduces to $(\mathbf{A}, \mathbf{B}) \equiv \langle \mathbf{A} \mathbf{B}^* \rangle$, where z^* denotes the complex conjugate of z . Clearly, the evaluation of this response function involves the time development of vector \mathbf{A} , which can formally be written in terms of the Liouville operator \mathcal{L} as

$$\mathbf{A}(t) = \exp(i\mathcal{L}t) \mathbf{A} = \sum_{j=0}^{\infty} \frac{t^j}{j!} \mathbf{A}^{(j)}. \quad (3)$$

\mathcal{L} is defined by the “diagonal matrix” $\mathcal{L} = \text{diag}(L, L, \dots, L)$. In the classical case, L represents the Poisson bracket with its Hamiltonian H and, in the quantal case, the corresponding commutator. In the following, we choose $\mathbf{A} = \mathbf{B}$; this involves no loss of generality as the matrix $\tilde{\chi}$ of interest can always be extracted from the response of $\mathbf{C} = (A_1 \cdots A_n, B_1 \cdots B_m)$ to an applied perturbation conjugate to \mathbf{C} . One notes $\tilde{C}_{AA}(t) = (\mathbf{A}(t), \mathbf{A})$ (all matrix quantities are denoted with a tilde).

Let us consider the Laplace representation of Eq. (1),

$$\tilde{\chi}(z) \tilde{\chi}^{-1}(0) = iz \tilde{a}_0(z) + \tilde{\Gamma}, \quad (4)$$

where $f(z) = \int_0^\infty e^{izt} f(t) dt$ is the Laplace transform of $f(t)$, -1 denotes matrix inversion, $\tilde{\Gamma}$ is the $n \times n$ unit matrix, $\tilde{\chi}(0) = \tilde{\chi}(z=0)$, and $\tilde{a}_0(z) = \tilde{C}_{AA}(z) \tilde{C}_{AA}^{-1}(t=0)$. The aim is to write down $\tilde{a}_0(z)$ in the CF form. Intuitively, noting the power series expansion of $\tilde{a}_0(z)$ in Eq. (3), a CF representation can be obtained by using the identity between a series and a CF [15]

$$F(x) = \alpha_0 x + \alpha_1 x^2 + \alpha_2 x^3 + \dots = \frac{\alpha_1 x}{1 + \frac{\alpha_2 x}{1 + \frac{\alpha_3 x}{1 + \dots}}}, \quad (5)$$

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which is satisfied if the Hankel determinants

$$H_n^1 = \begin{vmatrix} \alpha_0 & \alpha_1 & \cdots & \alpha_n \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n+1} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_n & \alpha_{n+1} & \cdots & \alpha_{2n} \end{vmatrix},$$

$$H_n^2 = \begin{vmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_{n+1} \\ \alpha_2 & \alpha_3 & \cdots & \alpha_{n+2} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{n+1} & \alpha_{n+2} & \cdots & \alpha_{2n+1} \end{vmatrix}$$

do not vanish. Note that $H_{-1}^1 = H_{-1}^2 = -1$, $a_1 = H_0^1$, and

$$a_{2n} = -\frac{H_{n-1}^2 H_{n-2}^1}{H_{n-2}^2 H_{n-1}^1}, \quad a_{2n+1} = -\frac{H_{n-2}^2 H_n^1}{H_{n-1}^2 H_{n-1}^1}.$$

The Lee approach that we generalize gives a transparent algorithm to obtain the CF recursively based on the following remarks. The correlation matrix $\tilde{C}_{AA}(t)$ may be interpreted as a ‘‘generalized scalar product’’ in the following sense. It satisfies the properties that define a scalar product but with complex numbers replaced by complex matrices and complex conjugation by Hermitian conjugation (dagger): $(\mathbf{A}, \mathbf{A}) \geq 0$, i.e., $(A_i, A_j) \geq 0 \forall i, j$; $(\mathbf{A}, \mathbf{A}) = 0$ if and only if $\mathbf{A} = (0, \dots, 0)$; $(\mathbf{A} + \mathbf{B}, \mathbf{C}) = (\mathbf{A}, \mathbf{C}) + (\mathbf{B}, \mathbf{C})$; $(\mathbf{A}, \mathbf{B}) = (\mathbf{B}, \mathbf{A})^\dagger$; $(\tilde{M}\mathbf{A}, \mathbf{B}) = \tilde{M}(\mathbf{A}, \mathbf{B})$ for any $n \times n$ complex matrix \tilde{M} . Moreover, operator \mathcal{L} is Hermitian in the sense that $(\mathbf{A}, \mathcal{L}\mathbf{B}) = (\mathcal{L}\mathbf{A}, \mathbf{B})^\dagger$, or equivalently $(\dot{\mathbf{A}}, \mathbf{B}) = -(\mathbf{B}, \dot{\mathbf{A}})$. Henceforth, having this strong analogy in mind, we will consider (\mathbf{B}, \mathbf{A}) as a scalar product on the vector space of n variables \mathbf{A} .

As seen in Eq. (3), $\mathbf{A}(t)$ evolves in the subspace spanned by the infinite set $\{\mathbf{A}^{(j)}\}$ of derivatives. In this subspace, one can construct an orthogonal basis $\{\mathbf{f}_n\}_{n=0,1,\dots}$ from the set $\{\mathbf{A}^{(j)}\}$, where orthogonality is understood as follows: vector \mathbf{X} is said to be orthogonal to vector \mathbf{Y} if (\mathbf{X}, \mathbf{Y}) is equal to the zero matrix $\tilde{0}$. Since we are interested in the scalar product $\tilde{C}_{AA}(t) = (\mathbf{A}(t), \mathbf{A})$, it is worth starting the recurrent process from the initial vector $\mathbf{f}_0 = \mathbf{A}$; \tilde{C}_{AA} will then correspond to the coefficient of \mathbf{A} in the expansion of $\mathbf{A}(t)$ on the orthogonal basis. The rest of the orthogonal basis is constructed inductively as in the Gram-Schmidt process and one obtains the recurrence relation

$$\mathbf{f}_j = \mathbf{A}^{(j)} - \sum_{k=0}^{j-1} (\mathbf{A}^{(j)}, \mathbf{f}_k) \cdot (\mathbf{f}_k, \mathbf{f}_k)^{-1} \mathbf{f}_k, \quad j \geq 1. \quad (6)$$

Therefore, $\mathbf{A}(t)$ can be expanded on the orthogonal set $\{\mathbf{f}_j\}$, such that $\mathbf{A}(t) = \sum_{k=0}^{\infty} \tilde{a}_k(t) \cdot \mathbf{f}_k$. Here, the coefficient $\tilde{a}_j(t)$'s are $n \times n$ matrices acting on the vectors \mathbf{f}_j 's. The boundary conditions are $\tilde{a}_0(t=0) = \tilde{1}$ and $\tilde{a}_j(t=0) = \tilde{0}$ for $j \geq 1$. The matrix $\tilde{a}_0(t)$ is the inverse Laplace transform of $\tilde{a}_0(z)$ in Eq. (4). The remaining $\tilde{a}_j(t)$'s can be related to memory functions defined by the generalized Langevin equation $\tilde{a}_j(t)$

$= (\mathbf{A}(t), \mathbf{f}_j) \cdot (\mathbf{f}_j, \mathbf{f}_j)^{-1}$. Taking the scalar product of $\mathbf{A}(t)$ with \mathbf{f}_j in Eq. (6) for each $j=0,1,\dots$, one obtains a recurrence relation between $\tilde{a}_0(t)$ and the \tilde{a}_j 's as well as the time derivatives $\tilde{a}_0^{(j)}(t)$ of $\tilde{a}_0(t)$

$$\tilde{a}_j(t) \tilde{D}_j = (-1)^j \tilde{a}_0^{(j)}(t) - \sum_{k=0}^{j-1} \tilde{a}_k \tilde{\Omega}_{jk}, \quad (7)$$

where the initial time derivatives $\tilde{a}_0^{(j)} = d^j \tilde{a}_0(0) / dt^j$ are calculated using the Hermitian character of \mathcal{L} , $\tilde{a}_0^{(j)}(t) = (-1)^j (\mathbf{A}(t), \mathbf{A}^{(j)}) \cdot (\mathbf{A}, \mathbf{A})^{-1}$. We have also defined the matrices $\tilde{D}_j = (\mathbf{f}_j, \mathbf{f}_j) \cdot (\mathbf{A}, \mathbf{A})^{-1}$ and $\tilde{\Omega}_{jk} = (\mathbf{A}^{(j)}, \mathbf{f}_k) \cdot (\mathbf{f}_k, \mathbf{f}_k)^{-1}$. In practice, as one goes further in the recurrent process, Eq. (7), reliable evaluations of \tilde{D}_j and $\tilde{\Omega}_{jk}$ are difficult, if not impossible. Moreover, the physical meaning of the higher-order matrices $\tilde{a}_j(t)$ is more and more obscure. In the following, we consider the first two steps in the construction of the CF. As we will see in the following, the second-order expression is suitable for reliable approximations.

By taking the Laplace transform of Eq. (7) for $j=1$, one obtains an expression of $\tilde{a}_0(z)$ in terms of a fraction

$$\tilde{a}_0(z) = [-iz\tilde{1} + \tilde{\Omega}_{10} + \tilde{b}_1(z)\tilde{\Delta}_1]^{-1}, \quad (8)$$

where $\tilde{b}_1(z) = \tilde{a}_0^{-1} \tilde{a}_1$ and $\tilde{\Delta}_1 = \tilde{D}_1 = (\mathbf{f}_1, \mathbf{f}_1) \cdot (\mathbf{f}_0, \mathbf{f}_0)^{-1}$. In the Mori approach, the matrix $\tilde{b}_1(z)$ is referred to as the memory function (here, memory matrix). An expression for $\tilde{b}_1(z)$ can be derived by using Eq. (7) for $j=2$ and one obtains the fraction

$$\tilde{b}_1(z) = (\tilde{\Omega}_{10}^2 - \tilde{\Omega}_{20})[-iz\tilde{1} + (\tilde{\Omega}_{21} - \tilde{D}_1\tilde{\Omega}_{10}) + \tilde{b}_2(z)\tilde{\Delta}_2]^{-1}, \quad (9)$$

where $\tilde{\Delta}_2 = \tilde{D}_2\tilde{D}_1^{-1} = (\mathbf{f}_2, \mathbf{f}_2)(\mathbf{f}_1, \mathbf{f}_1)^{-1}$. Further use of the recurrence relation Eq. (7) for $j=3,4,\dots$ would lead to the CF expression of the correlation matrix $\tilde{C}_{AA}(z)$. Note that, in order to facilitate the comparison with the case of a one-component dynamical variable, we have used the same notations for the orthogonal basis and the memory functions as in Refs. [7,16].

Application. As an illustration of the utility of the CF for a set of dynamical variables, we employ the formalism to model the Thomson scattering (TS) cross section of a dense two-component plasma (TCP) [17]. We consider a (neutral) strongly coupled TCP in equilibrium consisting of electrons e (charge $-e$, mass m_e , density n_e) and ions i (Ze , m_i , $n_i = n_e/Z$). An important quantity is the spectrum of the electron-density fluctuations, the so-called dynamic structure factor $S_{ee}(k, \omega)$, since it is directly probed in TS (here k and ω are, respectively, the wave vector and the frequency of the electron-density fluctuations) [18,19]. Within linear response theory, the classical fluctuation-dissipation theorem relates the dynamic structure factor to the electron-electron response function $\chi_{ee}(k, \omega)$

$$S_{ee}(k, \omega) = -\frac{2}{(n_e + n_i)\beta\omega} \text{Im} \chi_{ee}(k, z = \omega).$$

Our goal is to develop a CF representation for χ_{ee} and, therefore, for S_{ee} .

In view of the importance of the electron-ion coupling, we write the linear response of the TCP in terms of the vector

$$\mathbf{A}(k, t) = \begin{pmatrix} \delta n_e(k, t) \\ \delta n_i(k, t) \end{pmatrix}$$

of the Fourier components of the density fluctuations δn_e and δn_i . In this case, the general recurrence formulas, Eqs. (6) and (7), greatly simplify because the $\mathbf{A}^{(j)}$'s satisfy $(\mathbf{A}^{(\nu)}, \mathbf{A}^{(\mu)}) = \tilde{0}$ if $|\nu - \mu|$ is odd ($\nu, \mu \geq 0$). Equations (6) and (7) then have the simplified forms

$$\mathbf{f}_{j+1} = \mathbf{f}_j + \tilde{\Delta}_j \mathbf{f}_{j-1}, \quad (10)$$

$$\tilde{a}_{j+1}(t) \tilde{\Delta}_{j+1} = -\tilde{a}_j(t) + \tilde{a}_{j-1}(t), \quad (11)$$

where $j \geq 0$, $\mathbf{f}_{-1} = \mathbf{0}$, and $\tilde{\Delta}_{-1} = \tilde{1}$; here $\mathbf{f}_j = iL\mathbf{f}_j$, $\tilde{a}_j = d\tilde{a}_j/dt$, and $\tilde{\Delta}_j = (\mathbf{f}_j, \mathbf{f}_j) \cdot (\mathbf{f}_{j-1}, \mathbf{f}_{j-1})^{-1}$. Note that Eqs. (10) and (11) are the exact equivalents of the recurrence relations Eqs. (3) and (7) developed in Ref. [7] for a one-component dynamical variable.

These equations can be Laplace transformed to yield

$$\tilde{a}(k, z) = [-iz\tilde{1} + \tilde{b}_1(k, z)\tilde{\Delta}_1(k)]^{-1}, \quad (12)$$

$$\tilde{b}_1(k, z) = [-iz\tilde{1} + \tilde{b}_2(k, z)\tilde{\Delta}_2(k)]^{-1},$$

...

The CF representation of χ_{ee} is eventually obtained using Eq. (4). In Eq. (12), the coefficient matrices $\tilde{\Delta}_\nu(k)$ depend only on the static properties of the system through the sum rules $\langle \omega_{ab}^n(k) \rangle = \int (d\omega/2\pi) \omega^n S_{ab}(k, \omega)$. In particular, in matrix notation, $\tilde{\Delta}_1(k) = \langle \omega^2(k) \rangle \langle \omega^0(k) \rangle^{-1}$ and $\tilde{\Delta}_2(k) = \langle \omega^4(k) \rangle \langle \omega^2(k) \rangle^{-1} - \langle \omega^2(k) \rangle \langle \omega^0(k) \rangle^{-1}$. Thus, the determination of $\tilde{a}(k, z)$ is transferred to that of the matrices $\tilde{\Delta}_\nu$.

Having expressed the response function as a CF, a wide variety of approximations that exactly satisfy sum rules can be obtained. We first recall the concept of an effective interaction that is often described with a local field correction (LFC) in the one-component case. The frequency and wave-number dependent $\tilde{\chi}(k, \omega)$ can be exactly written down in terms of a reference system $\tilde{\chi}^0(k, \omega)$, usually taken as the free-particle system and a frequency and wave-vector dependent complex effective potential $\tilde{V}^{eff}(k, z)$ such that

$$\tilde{\chi} = \tilde{\chi}^0 (1 - \tilde{V}^{eff} \tilde{\chi}^0)^{-1}. \quad (13)$$

Here, $\tilde{\chi}^0$ corresponds to the response function matrix for a free-particle TCP and \tilde{V}^{eff} is a 2×2 symmetric matrix. The

effect of the strong electron-ion coupling on the TS cross section appears through the effective potential as

$$S_{ee}(k, \omega) = -\frac{2}{n\beta\omega} \text{Im} \left\{ \frac{\chi_{0,e}(k, \omega) [1 - V_{ii}^{eff}(k, \omega) \chi_{0,i}(k, \omega)]}{D(k, \omega)} \right\},$$

where

$$D = [1 - V_{ee}^{eff} \chi_{0,e}] [1 - V_{ii}^{eff} \chi_{0,i}] - (V_{ei}^{eff})^2 \chi_{0,e} \chi_{0,i}.$$

Using Eq. (12), one obtains

$$\begin{aligned} \tilde{V}^{eff}(k, z) &= \tilde{\chi}_0^{-1}(k, z) - \tilde{\chi}^{-1}(k, z) \\ &= \tilde{\chi}_0^{-1}(k) - \tilde{\chi}^{-1}(k) - iz \langle \omega^2(k) \rangle^{-1} [\tilde{b}_2(k, z) \tilde{\Delta}_2(k) \\ &\quad - \tilde{b}_{0,2}(k, z) \tilde{\Delta}_{0,2}(k)], \quad (14) \\ &= \tilde{\chi}_0^{-1}(k) - \tilde{\chi}^{-1}(k) - iz \langle \omega^2(k) \rangle^{-1} \\ &\quad \times \{ [-iz\tilde{1} + \tilde{b}_3(k, z) \tilde{\Delta}_3(k)]^{-1} \tilde{\Delta}_3(k) \\ &\quad - [-iz\tilde{1} + \tilde{b}_3^0(k, z) \tilde{\Delta}_3^0(k)]^{-1} \tilde{\Delta}_3^0(k) \}, \quad (15) \end{aligned}$$

for the interaction matrix \tilde{V}^{eff} . So far, we have obtained a formally exact representation for $S_{ee}(k, \omega)$ in terms of the effective interaction matrix \tilde{V}^{eff} . Although the CF form of Eq. (15) leads itself to a wide variety of approximations, it is important to ensure that basic properties of the correlation matrices are not violated. For example, matrix $\tilde{\chi}$ should be symmetric and matrix \tilde{a} should be positive semidefinite; otherwise the final result will be unphysical.

Let us now illustrate how various approximations can be constructed from Eq. (14). Obviously, an exact continuous fraction representation satisfies all the sum rules and only approximations will break them. The approximations introduced below are chosen to still exactly satisfy certain sets of these sum rules. The conventional approximation in many-body theory is the linearized Vlasov approximation [or random-phase approximation (RPA)] that corresponds to setting $\tilde{V}_{ab}^{eff}(k, \omega) = v_{ab}(k)$, where $v_{ab}(k)$ is an effective interaction between species a and b [20]. An improved approximation is the static local field correction (SLFC) $\tilde{V}^{eff}(k, \omega) = \tilde{V}^{eff}(k, 0)$, which can be viewed as setting $\tilde{b}_2 \tilde{\Delta}_2 = \tilde{b}_{0,2} \tilde{\Delta}_{0,2}$ in Eq. (14). This approximation guarantees that the sum rules $\langle \omega_{ab}^0(k) \rangle$, $\langle \omega_{ab}^2(k) \rangle$ are exactly satisfied. This particular approximation has been used previously in Ref. [21]. If we choose to satisfy $\langle \omega_{ab}^2(k) \rangle$ and $\langle \omega_{ab}^4(k) \rangle$, we are led to the high-frequency local field correction (HFLFC), $V_{ab}^{eff}(k, \omega) = V_{ab}^{eff}(k, \infty)$. In Ref. [17] we have shown that HFLFC compares favorably to molecular dynamics calculations than SLFC for strong coupling. A further step can be achieved by imposing the three sum rules $\langle \omega_{ab}^0(k) \rangle$, $\langle \omega_{ab}^2(k) \rangle$, and $\langle \omega_{ab}^4(k) \rangle$ to be satisfied. To this end, one could follow, for instance, the recipe given by Hong and Kim in Ref. [9] for a one-component plasma and extend it to encompass multicomponent systems. This recipe amounts to replacing the matrices $\tilde{b}_2(z), \tilde{b}_3(z), \dots$ in Eq. (15) with

their noninteracting counterparts $\tilde{b}_2^0(z), \tilde{b}_3^0(z), \dots$, respectively. For example, the approximation $\tilde{b}_2 = \tilde{b}_{f,2}$ can be calculated from $\tilde{\chi}_0(k, z)$ using Eqs. (4) and (12). However, this simple procedure can be shown to lead to an antisymmetric response function $\tilde{\chi}$.

These sum rules depend only on the pair distribution functions g_{ab} that, in practice, can be evaluated with a high degree of accuracy using the multicomponent hypernetted chain equations [22]. Note that higher-order sum rules depend on the n -body distribution functions ($n > 2$), which are nearly inaccessible in practical calculations. Thus, although the next approximation in the scheme described above, i.e., $\tilde{b}_3 = \tilde{b}_{f,3}$, would satisfy the four sum rules $n = 0, 1, 2$, and 3 , the $\langle \omega_{ab}^6(k) \rangle$'s must be determined only approximately, with, for example, a Kirkwood approximation [22]. Therefore, it seems that the truncation of the CF at the level containing \tilde{b}_2 , Eq. (14), is a good starting point for approximations.

As an illustration of the sensitivity of the results on the different approximations, we compare in Fig. 1 the dynamic structure factor $S_{ee}(k, \omega)$ for aluminum ($n_e = 10^{23} \text{ cm}^{-3}$, $T_i = T_e = 100 \text{ eV}$ and $Z = 3$) as obtained from calculations using the three approximations (RPA with semiclassical potentials [23], SLFC and HFLFC). In particular, one notes the differences between the approximations for the ion-acoustic feature (low frequency), showing the need for more refined models that can take into account the effect of strong coupling on the dynamical processes (collisions). The use of external parameters seems unavoidable. Whereas the first frequency-moment sum rules can be easily satisfied, these parameters will be directly linked to transports coefficients (such as the diffusion coefficient, viscosity, conductivity).

To summarize, we have generalized the Lee approach to

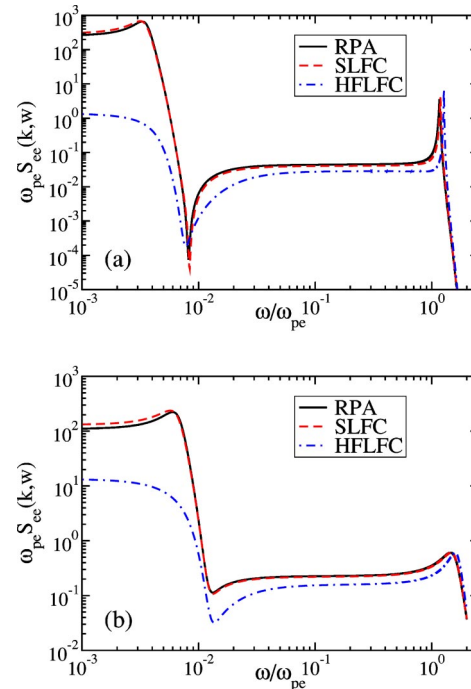


FIG. 1. Electron dynamic structure factor $S_{ee}(k, \omega)$ of aluminum ($n_e = 10^{23} \text{ cm}^{-3}$, $T_i = T_e = 100 \text{ eV}$ and $Z = 3$) plotted versus frequency normalized to ω_{pe} (double logarithmic scale) for (a) $k = 0.25a_i$ and (b) $k = 0.5a_i$ ($a_i = 3/4\pi n_i^{-3}$ is the ion-sphere radius).

the multiple dynamical variables case. A recurrence relation is derived that allows one to obtain the CF iteratively. To illustrate the utility of such modeling, we have applied the theory to the analysis of TS from strongly coupled two-component plasmas.

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